LOW-FREQUENCY CHARACTERISTIC ACOUSTIC OSCILLATIONS IN THE NONCIRCULATING PART OF A HYDRAULIC TURBINE

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In modern hydraulic turbines, there occur unsteady phenomena whose character are determined by the laws of acoustics. Theoretical investigation of these phenomena is in general a fairly difficult problem, due to the complex geometry of the noncirculating part of the turbine. Description of these phenomena is significantly simplified for low-frequency oscillations, making it possible to obtain a number of results of practical interest for engineering calculations. However, work in this area (for example, [1, 2]) is as a rule confined within the framework of a one-dimensional statement of the appropriate problem. In addition, because of its interaction with the turbine, both the steady and the unsteady components of the fluid flow becomes highly nonuniform.

In this work, the effect of twisted turbine flow on the character of the low-frequency hydroacoustic oscillations in the noncirculating part of the turbine is studied.

1. Fundamental Assumptions. We consider free hydroacoustic oscillations of the fluid in the noncirculating part of a hydraulic turbine. A diagram of the vertical cross section of the noncirculating part is shown in Fig. 1a (1 is the water intake pipe; 2 the volute chamber; 3 the discharge pipe), while Fig. 1b shows a horizontal cross section of the volute chamber in which the turbine is located. We introduce the natural coordinate system fixed to the line representing the geometric locus of the centers of the cross sections of the noncirculating part. The origin of the coordinate system is placed at the entrance to the discharge pipe section, and the positive direction of the axis is taken to be that of the fluid flow. As the characteristic geometric dimension, we take the length of the mean chord of a turbine blade b, and we assume that the following estimate is valid for the geometric dimensions of all other elements of the noncirculating part:

$$l_0 = O(b), \quad R_j = O(b) \quad (j = 1,\ 2); \tag{1.1}$$

$$b \ll l_2 \ll l_1, \tag{1.2}$$

where R_1, R_2 are the outer and inner radii of the turbine's working impeller; ℓ_1 is the length of the water intake pipe, ℓ_2 that of the discharge pipe; and ℓ_0 is the effective length of the volute chamber, whose value will be determined from the solution to the problem.

Assuming the cross-sectional areas of the intake and discharge pipes $\Omega(s)$ vary only slightly and reasonably smoothly, we take for simplicity

$$\Omega(s) = O(b^2) = \mathrm{const} \ \text{for} \ -(l_0 + l_1) < s < -l_0,\ 0 < s < l_2 \tag{1.3}$$

(s is the arc length coordinate of the noncirculating section).

We assume that the fluid is ideal, and that its motion is isentropic, and in addition, is irrotational in the intake pipe. We also assume that there are no body forces. Finally, assuming that the characteristic dimension determining the frequency of the characteristic oscillations is the length of the intake pipe, we write the oscillation frequency, in accordance with (1.2), as

$$k = \frac{\omega b}{c} = 2\pi \frac{b}{\lambda} \ll 1 \tag{1.4}$$

(ω is the circular frequency of oscillation; λ the wavelength; and c is the sound speed in the fluid).

2. Problem Statement. In the fluid flow region V (in the noncirculating section), we seek the characteristic frequencies and the characteristic functions of the free oscillations,

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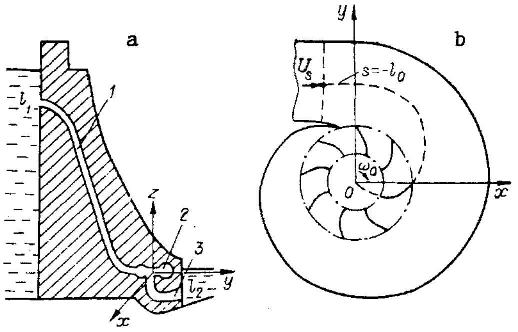

Fig. 1

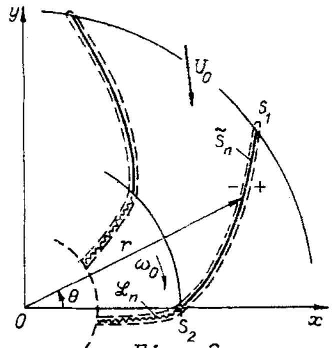

Fig. 2

which satisfy the following linearized system of equations based on the assumptions of Sec. 1:

$$\rho_0[\partial u'/\partial t+(\mathbf{U}_0\nabla)\mathbf{u}'+(\mathbf{u}'\nabla)\mathbf{U}_0]+\rho'(\mathbf{U}_0\nabla)\mathbf{U}_0=-\nabla p'; \tag{2.1}$$

$$\partial\rho'/\partial t+\nabla(\rho_0\mathbf{u}'+\rho'\mathbf{U}_0)=0; \tag{2.2}$$

$$\rho'=p'/c^2. \tag{2.3}$$

Here ρ_0, $\mathbf{U}_0$ are the steady, and ρ', $\mathbf{u}'$ the unsteady components of the density and velocity; and p' is the unsteady component of the pressure. In this case, ρ_0 and $\mathbf{U}_0$ are given quantities, and ρ', p', and $\mathbf{u}'$ are unknowns being sought in the form

$$\rho'=\rho e^{i\bar{\omega}t},\ p'=pe^{i\bar{\omega}t},\ \mathbf{u}'=\mathbf{u}e^{i\bar{\omega}t}, \tag{2.4}$$

where $\bar{\omega}=\omega(1+i\delta)$; δ is a parameter characterizing the stability of the oscillations.

The amplitude functions (2.4) must satisfy the following homogeneous boundary conditions:

impermeability of the solid-wall boundaries S, including the moving blade surfaces:

$$u_{\nu}=0 \text{ for } \boldsymbol{\rho}\in S \tag{2.5}$$

($\boldsymbol{\nu}$ is the direction of the normal to S, and $\boldsymbol{\rho}$ is the fluid particle radius vector);

at the open ends of the noncirculating section

$$p=0 \text{ for } s=-(l_0+l_1),\ l_2; \tag{2.6}$$

the Zhuskovsky–Kutta condition at the trailing edge of the blade c_n

$$[p]=0 \text{ for } \boldsymbol{\rho}\in c_n; \tag{2.7}$$

in the vortical wakes which trail the blade and are modeled by contact discontinuity surfaces $\mathscr{L}_n$:

$$[\mathbf{u}_{\nu_1}]=0,\ [p]=0 \quad\text{for}\quad \boldsymbol{\rho}\in\mathscr{L}_n \tag{2.8}$$

($\boldsymbol{\nu}_1$ is the direction of the normal to $\mathscr{L}_n$).

We seek an approximate solution of this problem. Toward this end, considering (1.4), we introduce the small parameter

$$k=\varepsilon. \tag{2.9}$$

To second-order accuracy $O(\varepsilon^2)$, the expressions for the characteristic functions in the intake and the discharge pipe can be given in analytical form. These expressions can be related to one another with the help of the integral laws of the conservation of mass and acoustical energy, applied to the fluid flow region in the volute chamber.

3. Representation of the Solution in the Intake Pipe. Bearing in mind (1.2) and (1.3), we introduce another assumption: the steady component of the fluid velocity in the intake pipe is assumed to have only a longitudinal component, and its modulus is taken to be

$$|\mathbf{U}_0|=U_s=\text{const}. \tag{3.1}$$

Also considering the assumption that the flow is irrotational in the intake pipe, the amplitude function of the unsteady component of the velocity is represented using a potential

$$\mathbf{u}_1=\nabla\varphi_1(s). \tag{3.2}$$

In this case, the pressure is also determined through the function φ_1 with the help of the Cauchy–Lagrange integral:

$$p_1 = -\rho_0(i\omega\varphi_1 + \mathbf{U}_0\nabla\varphi_1). \qquad (3.3)$$

Substituting (3.2), (3.3) into (2.1)-(2.3) and using (2.4), we find

$$(1 - M^2)\frac{\partial^2\varphi_1}{\partial s^2} - 2i\bar{k}M\frac{\partial\varphi_1}{\partial s} + \bar{k}^2\varphi_1 = 0.$$

Here and below the coordinate s and all geometric parameters are considered to be dimensionless quantities, by relating them to b:

$$M = U_s/c, \quad \bar{k} = \bar{\omega}b/c = k(1 + i\delta).$$

We take

$$M = O(\varepsilon), \quad \delta = O(\varepsilon). \qquad (3.4)$$

Assuming (without loss of generality) that $|\varphi_1/bc| \leqslant 1$, the solution to (3.3) satisfying condition (2.6) for $s = -(\ell_0 + \ell_1)$ can be represented in the following fashion:

$$\varphi_1 = Abce^{i\bar{k}M(s+l_0+l_1)}\left[\sin\bar{k}(s + l_0 + l_1) + iM\cos\bar{k}(s + l_0 + l_1)\right] \qquad (3.5)$$

[A is an arbitrary constant of order O(1)].

Substituting (3.5) into (3.2) and (3.3), we find

$$u_1 = A\bar{k}c\exp[i\bar{k}M(s + l_0 + l_1)]\cos\bar{k}(s + l_0 + l_1); \qquad (3.6)$$

$$p_1 = -iA\bar{k}c^2\exp[i\bar{k}M(s + l_0 + l_1)]\sin\bar{k}(s + l_0 + l_1). \qquad (3.7)$$

4. Representation of the Solution in the Volute Chamber. Unlike the fluid flow in the intake pipe, that in the volute chamber is highly nonuniform. We represent the unknown functions $\mathbf{u}'$, p', in the volute chamber as the sum of two components:

$$\mathbf{u}' = \mathbf{u}_0' + \mathbf{u}_\zeta', \quad p' = p_0' + p_\zeta',$$

each of which satisfies system (2.1)-(2.3). We assume that $\mathbf{u}_0'$, p_0' match up with the solution in the intake pipe at $s = -\ell_0$, i.e.,

$$u_{0s}' = u_1', \quad p_0' = p_1', \qquad (4.1)$$

and the condition that there be no circulation of the velocity $\mathbf{u}_0'$ about the contours L_n of the turbine blade profile

$$\oint_{L_n} \mathbf{u}_0' d\boldsymbol{\rho} = 0. \qquad (4.2)$$

In accordance with the overall statement of the problem, $\mathbf{u}_\zeta'$, p_ζ' will satisfy (2.5), (2.8) and, according to (2.7), the condition

$$[p_\zeta'] = [-p_0'] \quad \text{for} \quad \boldsymbol{\rho} \in c_n.$$

It follows from (4.1) that at the entrance to the volute chamber

$$u_{\zeta s}' = 0, \quad p_\zeta' = 0 \quad \text{for} \quad s = -l_0.$$

Since the fluid flow in the intake pipe is potential, then using Thompson's theorem on (4.1) and (4.2) gives $\nabla\times\mathbf{u}_0' = 0$. From this the following representation of $\mathbf{u}_0'$ is valid

$$\mathbf{u}_0' = \nabla\varphi_0' = \nabla\varphi_0 e^{i\bar{\omega}t}. \qquad (4.3)$$

We assume that the acoustic perturbations of the fluid created by the turbine and velocity potential φ_0' are equal at the entrance to the discharge pipe, so that

$$\varphi_0(\boldsymbol{\rho}) = \text{const}, \quad u_{0s}(\boldsymbol{\rho}) = \text{const}, \quad p_0(\boldsymbol{\rho}) = \text{const} \quad \text{for} \ s = 0.$$

The fluid flow described by the functions $\mathbf{u}_\zeta'$, p_ζ', is represented by the sum of purely circulational motion about the blading and motion induced by the unsteady vortical wakes that trail the blades. It can be shown that the flow has the property

$$\int_{S_2} u_{\zeta s}^{(0)} d\sigma = 0, \qquad (4.4)$$

where S_2 is the cross section at the joint with the discharge pipe; $u_{\zeta s}^{(0)}$ is the amplitude of the projection of the velocity $\mathbf{u}_{\zeta}'$ onto the direction s in the incompressible fluid approximation.

5. Representation of the Solution in the Discharge Pipe. For an analytical description of the flow in the discharge pipe, we introduce a cylindrical system of coordinate (r, θ, s), assuming that the pipe cross section is circular. Neglecting the flow in the radial direction, we will assume that the longitudinal component of the steady velocity satisfies condition (3.1), while the circumferential component is a function of the coordinate r only:

$$U_\theta = U_\theta(r).$$

Within the limits of these assumptions, system (2.1)-(2.3) is transformed in the following fashion:

$$\rho_0\left(\frac{\partial}{\partial t} + \mathbf{U}_0\cdot\nabla\right)\mathbf{u}' = -\nabla p'; \tag{5.1}$$

$$\frac{1}{c^2}\left(\frac{\partial}{\partial t} + \mathbf{U}_0\cdot\nabla\right)^2 p' = \Delta p'. \tag{5.2}$$

We now give the unknown functions $\mathbf{u}'$ and p' as the sum of two components:

$$\mathbf{u}' = \mathbf{u}_2'(s) + \mathbf{u}_w', \quad p' = p_2'(s) + p_w'.$$

Here $\mathbf{u}_2'$, p_2' are the velocity and pressure functions describing the one-dimensional acoustic oscillations of the fluid in the discharge pipe; and $\mathbf{u}_w'$, p_w' are the corresponding functions describing the wave motion induced by the vortical wakes.

We will limit our discussion to mean integral values over the cross section S(s) of the amplitude functions:

$$\tilde{p}_w = \frac{1}{\Omega}\int_S p_w d\sigma, \quad \tilde{u}_{ws} = \frac{1}{\Omega}\int_S u_{ws} d\sigma, \quad \tilde{u}_{w\theta} = \frac{1}{\Omega}\int_S u_{w\theta} d\sigma.$$

Integrating Eq. (5.2) over S, we obtain

$$(1 - \mathrm{M}^2)\frac{\partial^2\tilde{p}}{\partial s^2} - 2i\bar{k}\mathrm{M}\frac{\partial\tilde{p}}{\partial s} + \bar{k}^2\tilde{p} = 0,$$

where $\tilde{p} = p_2 + \tilde{p}_w$. We represent the solution of this equation satisfying condition (2.6) for $s = \ell_2$ as:

$$\tilde{p} = -iB\bar{k}c^2\rho_0 e^{i\bar{k}\mathrm{M}(s-l_2)}\sin\bar{k}(s - l_2) \tag{5.3}$$

(B is an arbitrary constant).

Without losing generality, we assume

$$p_2 = \tilde{p}, \quad \tilde{p}_w = 0. \tag{5.4}$$

We determine the one-dimensional function u_2' as a particular solution of (5.1), the right-hand side of which is expressed in terms of the function p_2'. As a result, the expression for the amplitude function u_2 takes the form

$$u_2 = B\bar{k}c\left[e^{i\bar{k}\mathrm{M}(s-l_2)}\cos\bar{k}(s - l_2) + O(\varepsilon^3)\right]. \tag{5.5}$$

It is clear that one-dimensional acoustic oscillations of the fluid described by the functions u_2 and p_2 are potential and the corresponding amplitude functions of the velocity potential can be represented in the following way:

$$\varphi_2 = Bcbe^{i\bar{k}\mathrm{M}(s-l_2)}\left[\sin\bar{k}(s - l_2) + i\mathrm{M}\cos\bar{k}(s - l_2)\right]. \tag{5.6}$$

Integrating (5.1) over the cross section S and taking (5.4) into account, we find

$$\left(\frac{\partial}{\partial t} + U_s\frac{\partial}{\partial s}\right)\tilde{\mathbf{u}}_w' = 0.$$

From this we have

$$\tilde{u}_{ws} = C\bar{k}ce^{-i\frac{\bar{\omega}}{U_s}s}, \quad \tilde{u}_{w\theta} = D\bar{k}ce^{-i\frac{\bar{\omega}}{U_s}s}$$

(C and D are arbitrary constants).

Recall that the functions $\tilde{u}_{ws}$ and $\tilde{u}_{w\theta}$ describe the fluid wave motion induced by the unsteady vortical wakes. Bearing in mind property (4.4) for the corresponding motion in the volute chamber, we obtain the estimate $C = O(\varepsilon^2)$ for the constant C. The constant D characterizes the unsteady twisted flow and can be determined with the help of theorems on the change in momentum in the fluid as a result of its interaction with the turbine. Thus, the conditions that the solutions in the discharge pipe join with that in the volute chamber can be represented in the form

$$u_2 = u_0[1 + O(\varepsilon^2)], \quad p_2 = p_0 + \tilde{p}_\zeta \text{ for } s = 0, \tag{5.7}$$

where $\tilde{p}_\zeta$ is the mean integral value of the amplitude function for the pressure component p'_ζ in the volute chamber, coupled with the twisted flow in the pipe. The constant B for the functions u_2 and p_2 and the constant A for the functions u_1 and p_1 are determined from matching conditions, which as noted above, we will construct with the help of conservation laws.

6. The First Matching Condition. Integrating Eq. (2.2) over the fluid flow volume in the volute chamber V_0 and applying the Gauss–Ostrogradskii theorem, we obtain

$$\frac{1}{c^2}\frac{\partial}{\partial t}\int_{V_0} p' dv = -\oint_{S_0}\left(\rho_0 \mathbf{u}' + \frac{1}{c^2}\mathbf{U}_0 p'\right)\cdot \mathbf{v} d\sigma \tag{6.1}$$

($\mathbf{v}$ is the outer normal to the boundary S_0 of the region V_0).

First we consider the volume integral in expression (6.1). Toward this end, we estimate the order of magnitude of the functions $\mathbf{u}'$ and p' in the region V_0. According to (3.6), (5.5), and (3.4), we have

$$|\mathbf{u}'| = cO(\varepsilon), \quad p' = \rho_0 c^2 O(\varepsilon). \tag{6.2}$$

It follows from (2.1) and (2.2) that

$$|\nabla \mathbf{u}'| = cO(\varepsilon^2), \quad |\nabla p'| = \rho_0 c^2 O(\varepsilon^2); \tag{6.3}$$

$$|\Delta \mathbf{u}'| = cO(\varepsilon^3), \quad \Delta p' = \rho_0 c^2 O(\varepsilon^3). \tag{6.4}$$

Taking (6.2)-(6.4) into account, we find

$$\frac{\partial}{\partial t}\int_{V_0} p' dv = i\omega\frac{V_0}{2}\left[(p_1' + p_2') + \rho_0 c^2 O(\varepsilon^3)\right], \tag{6.5}$$

where p_1', p_2' are the unsteady components of the pressure in the intake pipe and the discharge pipe at the joint sections with the volute chamber; their amplitude functions are determined by (3.7) and (5.3).

To compute the integral over S_0 in (6.1), it must be borne in mind that part of S_0 is the surface $\tilde{S}_n$ of the rotating turbine blades. Moving around $\tilde{S}_n$ as depicted schematically in Fig. 2 and assuming that at the boundary of the volute chamber $\mathbf{U}_0\cdot\mathbf{v} = \mathbf{u}'\cdot\mathbf{v} = 0$, and at the blade surface $\mathbf{u}'\cdot\mathbf{v} = 0$, we represent this integral in the form

$$J_{S_0} = \sum_{n=1}^{N}\oint_{\tilde{S}_n}\frac{p'}{c^2}\mathbf{U}_0\cdot\mathbf{v} d\sigma + \left(\int_{S_2} - \int_{S_1}\right)\left(\rho_0 u_s' + \frac{M}{c}p'\right) d\sigma \tag{6.6}$$

(S_1 and S_2 are the cross sections of the volute chamber at the joint with the intake pipe and the discharge pipe, respectively).

In the case of uniform blading, the integrals over the surfaces of all blades will be the same. Applying for simplicity the hypothesis of plane cross sections and modeling the blades as impervious surfaces, we reduce these integrals to an iterated integral

$$J_{\tilde{S}_n} = \int_0^h \oint_L p'\mathbf{U}_0\cdot\mathbf{v} d\sigma dz \quad (n = 1, 2, \ldots, N) \tag{6.7}$$

[h(s) is the blade height]. To compute integral (6.7), we represent $\mathbf{U}_0$ in the form $\mathbf{U}_0 = \mathbf{U}_r + \mathbf{U}_e$ ($\mathbf{U}_r$ is the relative vector velocity in the coordinate system rigidly fixed to the rotating turbine; $\mathbf{U}_e$ is the translational velocity). Bearing in mind that

$$\mathbf{U}_r\cdot\mathbf{v} = 0, \quad \mathbf{U}_e\cdot\mathbf{v} = (\boldsymbol{\omega}_0\times\mathbf{r})\cdot\mathbf{v} = \omega_0 \mathbf{r}\sin\alpha$$

($\boldsymbol{\omega}_0$ is the vector angular rotation rate of the turbine; $\mathbf{r}$ is the radius vector of a point on the surface contour, relative to points lying on the axis of rotation; and α is the angle between the vectors $\mathbf{r}$ and $\mathbf{v}$), we find

$$J_L = \oint_L p' \mathbf{U}_0 \cdot \mathbf{v} d\sigma = \omega_0 \int_{s_1}^{s_2} (p'^- - p'^+) r \sin \alpha ds. \tag{6.8}$$

Here s_1 and s_2 are the coordinates of the leading and trailing edges of the profile; the plus superscripts indicates that the appropriate value of p' is taken at the weather side of the surface, while the minus superscripts indicates that it is taken on the lee side.

It is not difficult to see that integral (6.8) determines the unsteady components of the hydrodynamic torque acting on the profile, with respect to the rotational axis of the blading. Taking this into account, we represent the first term in (6.6) as

$$\sum_{n=1}^{N} \oint_{\tilde{S}_n} \frac{p'}{c^2} \mathbf{U}_0 \cdot \mathbf{v} d\sigma = \frac{\omega_0}{c^2} \int_0^h M'_Q dz = \rho_0 \mathrm{M} \frac{\omega_0}{c} R_2 \Omega m_Q u'_2, \tag{6.9}$$

where M'_Q is the hydrodynamic torque per unit length acting on the turbine blade as a result of the oscillations of the fluid discharge according to the law $Q = \rho_0 \Omega u'_2$; and m_Q is the dimensionless complex coefficient of the torque.

Over the joint cross section S_j, the integral (6.6) has the form

$$J_{S_j} = \Omega \left(\rho_0 u'_{js} + \frac{\mathrm{M}}{c} p'_j \right) \quad (j = 1, 2). \tag{6.10}$$

Substituting (6.5) and (6.6) into (6.1), and using (6.9) and (6.10), we obtain

$$u_1 + \frac{1}{\rho_0 c}\left(\mathrm{M} - i\bar{k}\frac{V_0}{2\Omega b}\right) p_1 = \left(1 + \mathrm{M}\frac{\omega_0 R_2}{c} m_Q\right) u_2 + \frac{1}{\rho_0 c}\left(\mathrm{M} + i\bar{k}\frac{V_0}{2\Omega b}\right) p_2 + cO(\varepsilon^4). \tag{6.11}$$

We consider this relation as the first matching condition for the unknown functions determining the acoustic oscillations in the water intake and discharge pipes.

7. The Second Matching Condition. We obtain the second matching condition with the help of the law of conservation of acoustic energy, which we will apply to the acoustic oscillations described by the function φ'_0 in the region V_0. In accordance with [3] and the assumptions that have been made, the conservation law in integral form is

$$\frac{\partial}{\partial t} \int_{V_0} E dv + \int_{S_0} \mathbf{I} \cdot \mathbf{v} d\sigma = V_0 c^2 O(\varepsilon^3), \tag{7.1}$$

where E, I are the density and the vector flux intensity of the acoustic energy:

$$E = \frac{1}{2}\left(\frac{p_0'^2}{c^2} + \rho_0 \mathbf{u}_0'^2\right) + \frac{p'_0}{c^2}\mathbf{u}'_0 \cdot \mathbf{U}_0, \quad \mathbf{I} = \left(\frac{p'_0}{\rho_0} + \mathbf{u}'_0 \cdot \mathbf{U}_0\right)\left(\rho_0 \mathbf{u}'_0 + \frac{p'_0}{c^2}\mathbf{U}_0\right).$$

For the amplitude functions of the unsteady components of the flow parameters, we use (4.3) to transform relation (7.1) to the following form of order $V_0 c^2 O(\varepsilon^2)$:

$$J = \int_{V_0} \left[-\frac{k^2}{b^2}\varphi_0^2 + (\nabla\varphi_0)^2 \right] dv - \int_{S_0} \varphi_0 \left[\nabla\varphi_0 - \frac{ik\mathbf{U}_0}{bc}\varphi_0 \right] \mathbf{v} d\sigma = 0. \tag{7.2}$$

First we consider the volume integral in (7.2). Since the function φ_0 is unknown, this integral, like (6.5), will be determined by the fluid flow parameters in the intake and discharge pipes in the joining sections S_j using (4.1) and (5.7). It should be noted that $\tilde{p}_\zeta$ in relation (5.7) is unknown. To find it, we apply the law of conservation of mass to the acoustic oscillations described by the function φ'_0 in the region V_0. Proceeding as in Sec. 6, we obtain

$$u_1 + \frac{1}{\rho_0 c}\left(\mathrm{M} - i\bar{k}\frac{V_0}{2\Omega b}\right) p_1 = u_2 + \frac{1}{\rho_0 c}\left(\mathrm{M} + i\bar{k}\frac{V_0}{2\Omega b}\right) p_0 + \frac{\omega_0}{\rho_0 c^2} M_0 + cO(\varepsilon^3), \tag{7.3}$$

where M_0 is the amplitude of the unsteady component of the hydrodynamic torque acting on the blading with condition (4.2). The value of M_0 can be determined with the help of the theorem on the change in fluid momentum in the volute chamber. Expressing this change in terms of the flow parameters at the volute chamber entrance and exit, and bearing in mind that in the present case $\mathbf{u}_0 = 0$ when s = 0, we find

$$M_0 = -\rho_0 u_2 [1 + O(\varepsilon)] \int\limits_{S_2} U_\theta r d\sigma. \quad (7.4)$$

Computing (7.3) from (6.11) and using (5.7), (7.4), we have

$$\tilde{p}_\zeta (\mathrm{M} + ik\bar{V}_0) = -\rho_0\omega_0 R_2 u_2 (\mathrm{M} m_Q + m_z(\mathrm{U}_0)),$$

where

$$\bar{V}_0 = V_0/2\Omega b; \quad m_z(\mathrm{U}_0) = \frac{1}{cR_2\Omega}\int\limits_{S_2} U_\theta r d\sigma.$$

We now write the adhesion condition for φ_0 when s = 0:

$$\varphi_0(0) = \frac{i}{\omega}\left(\frac{p_2 - \tilde{p}_\zeta}{\rho_0} + u_2 U_s\right) = \varphi_2(0) - i\frac{\tilde{p}_\zeta}{\rho_0\omega}. \quad (7.5)$$

Using (7.5) and proceeding as in Sec. 6, we obtain from (7.2) the expression

$$\left[\varphi_1\left(u_1 + \frac{\mathrm{M}}{\rho_0 c}p_1\right) + \bar{V}_0 b\left(-\frac{k^2}{b^2}\varphi_1^2 + u_1^2\right)\right]_{s=-l_0} = \left[\varphi_2\left(u_2 + \frac{\mathrm{M}}{\rho_0 c}p_2\right) - \bar{V}_0 b\left(-\frac{k^2}{b^2}\varphi_2 + u_2^2\right) - i\frac{\tilde{p}_\zeta}{\rho_0\omega}u_2\right]_{s=0}, \quad (7.6)$$

which we regard as the second matching condition for the unknown functions.

8. Eigenvalues and Eigenfunctions. Substituting (3.5)-(3.7), (5.3)-(5.6) into (6.11) and (7.6), we find

$$A\exp(i\bar{k}\mathrm{M}(l_1 + l_2))\,[\cos\bar{k}l_1 - (\bar{k}\bar{V}_0 + i\mathrm{M})\sin\bar{k}l_1] = B\,[\cos\bar{k}l_2 + i\mathrm{M}\sin\bar{k}l_2 - \bar{k}\bar{V}_0\sin\bar{k}l_2]; \quad (8.1)$$

$$A^2\exp(2i\bar{k}\mathrm{M}(l_1 + l_2))\left[\frac{1}{2}(1 + \mathrm{M}^2)\sin 2\bar{k}l_1 + \left(i\mathrm{M} + \bar{k}\bar{V}_0\cos 2\bar{k}l_1\right)\right] =$$
$$= -B^2\left[(\cos\bar{k}l_2 + i\mathrm{M}\sin\bar{k}l_2)(\sin\bar{k}l_2 - i\mathrm{M}\cos\bar{k}l_2) + \bar{k}\bar{V}_0\cos 2\bar{k}l_2 - \bar{k}\Phi\cos^2\bar{k}l_2\right], \quad (8.2)$$

where

$$\Phi = \Phi' + i\Phi'' = i\frac{\omega_0 R_2}{\omega b}\,\frac{m_Q\mathrm{M} + m_z(\mathrm{U}_0)}{\mathrm{M} + i\bar{k}\bar{V}_0}.$$

Squaring the left- and right-hand sides of (8.1), we have, in conjunction with (8.2), a homogeneous system of algebraic equations in terms of the unknowns A^2 and B^2. From the condition that the determinant of the system equal zero, we find the complex eigenvalues for this problem to order $O(\varepsilon^2)$:

$$\bar{k}_m = \frac{m\pi}{l_0 + l_1 + l_2}\left(1 + i\frac{\cos^2 kl_2}{l_1 + l_2}\Phi''\right) \quad (m = 1, 2, \ldots); \quad (8.3)$$

$$\bar{k}_{jn} = \frac{\pi}{l_j + \bar{V}_0}\left[\left(\frac{1}{2} + n\right) + i(-1)^j\frac{\mathrm{M}}{\pi}\right] \quad (j = 1,2;\ n = 0, 1, 2, \ldots). \quad (8.4)$$

Here $\ell_0 = 2\bar{V}_0 - \Phi'\cos^2 k\ell_2$ is the derived effective length of the volute chamber. Using (8.3) and (8.4), we obtain the following relations from (8.1):

$$B_m = (-1)^m\exp(i\bar{k}\mathrm{M}(l_1 + l_2))\left(1 + \frac{2\bar{V}_0 + \Phi\cos^2 kl_2}{l_1 + l_2}\right)A_m, \quad B_{1n} = A_{2n} = 0, \quad (8.5)$$

substituting these into (3.5) and (5.6), we find expressions for the eigenfunctions in the intake and discharge pipes.

9. Results and Discussion. The set of eigenvalues obtained, whose domain of definition is limited by condition (1.3), can be divided into three subsets:

$\{\bar{k}_m\}$ are those eigenvalues determining the characteristic frequencies of the fluid oscillations in the circulating section, whose total length ℓ is multiplied by half of the corresponding wavelength:

$$l = l_0 + l_1 + l_2 = m\frac{\lambda_m}{2} \quad \left(\lambda_m = cT_m = \frac{2\pi b}{k_m}\right);$$

$\{\bar{k}_{jn}\}$ are those eigenvalues determining the characteristic frequencies of the fluid oscillations in the water intake (j = 1) and discharge (j = 2) pipes, whose lengths, constructed serially from the effective half-length of the volute chamber, are multiplied by one quarter of the corresponding wavelengths:

$$l_j + \overline{V}_0 = (1 + 2n)\frac{\lambda_{jn}}{4} \quad \left(\lambda_{jn} = \frac{2\pi b}{k_{jn}}\right).$$

It follows from (8.5) that the unsteady components of the flow parameters corresponding to the elements of subset $\{\bar{k}_m\}$ are discontinuous functions of s. In this case the amplitude of the flow velocity oscillations near the turbine is close to its maximum value. For those oscillations corresponding to elements of the subset $\{\bar{k}_{jn}\}$, the amplitude of the velocity oscillations near the turbines is equal to zero, while the unsteady components of the pressure undergo a jump across the turbine. The latter condition indicates that these oscillations cannot be realized separately, since the jump in the unsteady pressure component can arise only during unsteady circulational flow, which in turn is proportional to the amplitude of the fluid oscillations near the turbine. They are engendered by the oscillations corresponding to elements of the subset $\{\bar{k}_m\}$ and are produced together with them as a unit. The mechanism whereby oscillations corresponding to $\{\bar{k}_{jn}\}$ arise is evidently related to the nonlinear interaction of the turbine with the unsteady flow, whose description is implicitly contained in the second matching condition. Thus we call such oscillations quasicharacteristic. Their existence is supported by the results of experiments carried out by Arm [4]. The fluid oscillation frequencies in the circulating section of the hydraulic turbine, which result from spectral analysis of natural phenomenon in [4] agree to order $cO(\varepsilon)/b$ with frequencies ω_1 (m = 1), ω_{10} computed from (8.3), (8.4). The experimentally established fact of steep growth in the oscillation intensity near conditions of maximum fluid discharge also agrees with the theoretical results obtained here. According to (8.3), the observed phenomenon can be interpreted as oscillational instability, for which the following inequality serves as a condition

$$\Phi'' = \mathrm{M}\frac{\omega_0 R_2}{\omega b}\frac{m_z(\mathrm{U}_0) + k\overline{V}_0 m_Q'' + \mathrm{M} m_Q'}{\mathrm{M}^2 + k^2\overline{V}_0^2} < 0,$$

since first, under these conditions $m_z(\mathrm{U}_0) < 0$, and second, according to the theory of blading in unsteady flow, in separating and near-separating conditions there is a high probability of a change in sign from positive to negative in both the imaginary and real parts of the coefficient of the unsteady component of the momentum m_Q. It should be noted that the theoretical determination of the coefficients for m_Q is quite complex, even for unseparated blading flow [5, 6]. Experimental results obtained for axial blading [7] give some representation of the behavior of the unsteady hydrodynamic characteristics in separated flow regimes.

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